



Cambridge International AS & A Level

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MATHEMATICS

9709/12

Paper 1 Pure Mathematics 1

February/March 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

1 A line has equation $y = 3x - 2k$ and a curve has equation $y = x^2 - kx + 2$, where k is a constant.

Show that the line and the curve meet for all values of k . [4]

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2 A function f is defined by $f(x) = x^2 - 2x + 5$ for $x \in \mathbb{R}$. A sequence of transformations is applied in the following order to the graph of $y = f(x)$ to give the graph of $y = g(x)$.

Stretch parallel to the x -axis with scale factor $\frac{1}{2}$

Reflection in the y -axis

Stretch parallel to the y -axis with scale factor 3

Find $g(x)$, giving your answer in the form $ax^2 + bx + c$, where a , b and c are constants. [4]

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- 3 A curve has equation $y = \frac{1}{60}(3x + 1)^2$ and a point is moving along the curve.

Find the x -coordinate of the point on the curve at which the x - and y -coordinates are increasing at the same rate. [4]

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4 The circumference round the trunk of a large tree is measured and found to be 5.00 m. After one year the circumference is measured again and found to be 5.02 m.

- (a) Given that the circumferences at yearly intervals form an arithmetic progression, find the circumference 20 years after the first measurement. [2]

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- (b) Given instead that the circumferences at yearly intervals form a geometric progression, find the circumference 20 years after the first measurement. [3]

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5 Points $A(7, 12)$ and B lie on a circle with centre $(-2, 5)$. The line AB has equation $y = -2x + 26$.

Find the coordinates of B . [6]

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7 (a) By first obtaining a quadratic equation in $\cos \theta$, solve the equation

$$\tan \theta \sin \theta = 1$$

for $0^\circ < \theta < 360^\circ$.

[5]

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(b) Show that $\frac{\tan \theta}{\sin \theta} - \frac{\sin \theta}{\tan \theta} \equiv \tan \theta \sin \theta$. [3]

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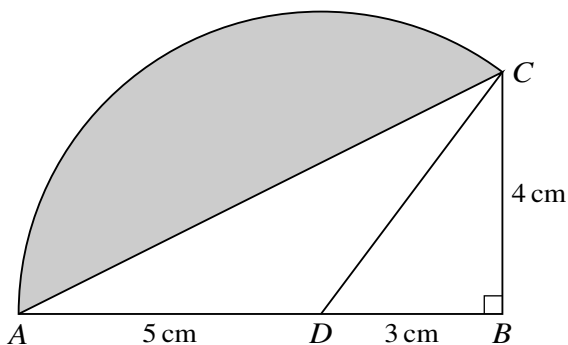
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The diagram shows triangle ABC in which angle B is a right angle. The length of AB is 8 cm and the length of BC is 4 cm. The point D on AB is such that $AD = 5$ cm. The sector DAC is part of a circle with centre D .

(a) Find the perimeter of the shaded region.

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(b) Find the area of the shaded region.

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9 The function f is defined by $f(x) = -3x^2 + 2$ for $x \leq -1$.

(a) State the range of f . [1]

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(b) Find an expression for $f^{-1}(x)$. [3]

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The function g is defined by $g(x) = -x^2 - 1$ for $x \leq -1$.

(c) Solve the equation $fg(x) - gf(x) + 8 = 0$.

[5]

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10 At the point $(4, -1)$ on a curve, the gradient of the curve is $-\frac{3}{2}$. It is given that $\frac{dy}{dx} = x^{-\frac{1}{2}} + k$, where k is a constant.

(a) Show that $k = -2$. [1]

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(b) Find the equation of the curve. [4]

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(c) Find the coordinates of the stationary point. [3]

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(d) Determine the nature of the stationary point. [2]

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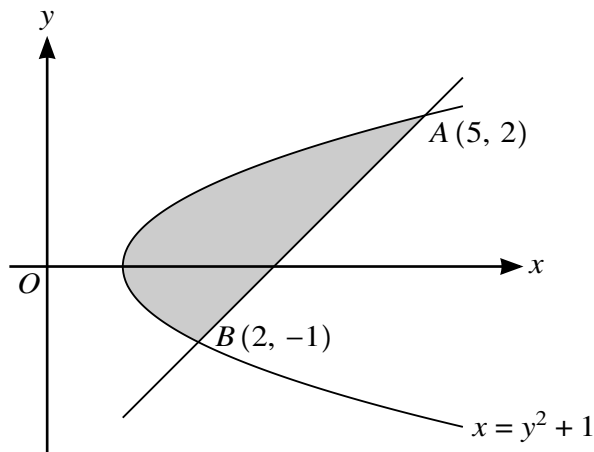
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The diagram shows the curve with equation $x = y^2 + 1$. The points $A(5, 2)$ and $B(2, -1)$ lie on the curve.

- (a) Find an equation of the line AB . [2]

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- (b) Find the volume of revolution when the region between the curve and the line AB is rotated through 360° about the y -axis. [9]

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Additional Page

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